Our Universe: The Math Behind the Motion

Kellan Glass

Beaver Country Day School

Cohn Grant Project

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One of the oldest questions physicists have ever asked themselves is surprisingly simple: why do planets stay in orbit? If what we know about gravity is true, that it pulls objects towards one another, then shouldn't the Earth, along with everything else, fall into the sun? At the same time, if planets are constantly moving throughout space, why don't they simply fly away and escape the sun's gravity? The answer lies in a balance between a planet's motion and the gravitational force acting on it. Gravity pulls a planet inward, while the planet's velocity carries it forward.[1] The result of this is neither a collision nor an escape, but rather a path, dependent on the characteristics of the planet, which we call an orbit. To describe this phenomenon mathematically, physicists derive what is known as the orbit equation. Starting from Newton's laws of motion and gravitation, the orbit equation provides a single mathematical equation that describes the path of two bodies interacting through gravity in an ideal Newtonian model. The equation is extraordinary because it encompasses circular, elliptical, parabolic, and hyperbolic trajectories, showing that they are all solutions to the same physical laws.[2] Because of this, the equation of the orbit is one of the most important equations in mathematics. Although this raises the question: how does a single equation describe nearly every orbit in the Solar System, and what are its limitations? Newton's Law of Gravitation, together with Newton's Laws of Motion and the conservation of angular momentum, leads directly to the orbit equation. This equation explains why planets, moons, satellites, and comets all follow conic-section trajectories, although it still has limitations.

Before we had the math to describe it, many famous astronomers observed planetary motion for hundreds of years. In the early 1600s, Johannes Kepler was trying to find the actual paths planets followed around the Sun. Among those whose findings aided Kepler, Tycho Brahe provided some of the most precise measurements of planetary positions before the telescope. During this period, many believed planets followed perfect circles. However, as Kepler compared Brahe's measurements to this assumption, he eventually found that the data was contradictory. After years of work, Kepler concluded that the problem was not in the observations, but in the assumption that orbits had to be circular.[3] From this work, Kepler began to develop his three laws of planetary motion, known today

as Kepler’s Laws. His first law states that planets move in elliptical orbits, with the Sun at one focus. His second law states that a line connecting a planet to the Sun sweeps out equal areas in equal times, meaning that a planet moves faster when it is closer to the Sun and slower when it is farther away. His third law established a relationship between an orbit's size and the time required to complete it.[4] For the time, Kepler’s Laws worked incredibly well because they were based on years of observations, rather than statements without backing. They were as close to a mathematical model that describes planetary motion as ever before. That said, Kepler discovered the patterns of the motions. However, Kepler’s laws were still descriptive, as they were based completely on observations. They could describe the planet’s orbit and how its speed changed, but they could not explain the physical cause. Kepler discovered these laws before Newton’s theory of universal gravitation. NASA says that Kepler himself was unaware that gravity was the acting force holding the planets in orbit.[5] That explanation came later with Isaac Newton, who showed that Kepler's laws were consequences of his laws of motion.[6] This distinction is important for understanding Isaac Newton's orbit equation. Kepler provided the observations, while Newton provided the reason, and the orbital equation connects the two.

While Kepler’s laws did an excellent job describing planetary motion, they left a key aspect out of their description: what causes planets to move the way they do? Isaac Newton sought to answer this by connecting the motion of objects on Earth to that of objects in space, arguing that they could be driven by the same force: gravity.[7] This observation was the first major step toward the equation of motion and a fundamental understanding of our universe. Newton continued to explore gravity and eventually concluded that it is universal, meaning every object with mass attracts every other object with mass. The strength of this attraction depends on the mass of the two objects and decreases depending on the distance between them. This relationship is defined by the inverse square law, which is especially important in planetary motion. As a planet gets further from the Sun, the gravitational effect the Sun has on that planet weakens, and vice versa.[8] Newton expressed this mathematically as $F = GMm/r^2$. One way of understanding what Newton discovered is through the falling moon idea. Newton recognized that the Moon is not outside the influence of Earth’s gravity. Instead,

the Moon constantly accelerates toward Earth. If gravity were to hypothetically disappear suddenly, the Moon's speed would carry it in an approximately straight line, but because of Earth's gravity, it curves the Moon's trajectory, creating an orbit. In this sense, we can describe an orbiting object as continuously falling without ever reaching the surface.[9] When comparing the numbers, it is proven that the gravitational acceleration predicted at the Moon's distance is the same as the acceleration needed to keep the Moon in orbit. Or in other words, the same gravity that keeps the Moon in orbit is the same gravity that makes an apple fall. Newton's theory provided the explanation that Kepler's laws missed, or, moreover, explained why things happen the way they do. Newton therefore changed the study of planetary motion from a description to a physical and mathematical theory. Kepler showed how planets moved; Newton showed why they move.[10] This connection between force and motion also serves as the starting point for Newton's derivation of the Orbit Equation.

Kepler had shown that planets move in predictable ways, which Newton used to prove with physical laws. The key idea is relatively simple: a planet moves through space while gravity continuously pulls it toward the Sun. The orbit equation describes the result of these two effects. The most important part of the derivation, and the starting point, is Newton's law of gravitation. This is the same equation previously mentioned, and is represented as $F = GMm/r^2$. Here, M and m are the masses of the two objects, r is the distance between them, and G is the gravitational constant.[11] The most important part of this equation is the r^2 in the denominator. Gravity follows the inverse-square law, meaning that as two objects move farther apart, the gravitational attraction between them decreases rapidly. For example, if you were to double the distance, the attraction would be one-fourth as strong.[12] Gravity is also known as a central force. The Sun does not pull objects downward; it pulls objects directly toward itself. That said, as the planet moves, the direction of the force changes. This continuous gravitational pull is what turns an otherwise straight-line trajectory into an orbit.[13] Up until this point, we have only referenced the Sun and one planet, although the solar system consists of the Sun, eight planets, hundreds of moons, and countless smaller objects. All of these objects' gravity constantly acts on each other. If we were to include every single one of these objects in a calculation, it would take

what could be a simple derivation and make it almost impossible in some cases. The first approximation is to only use two bodies, such as the Moon and Earth. This approximation is known as the two-body problem.[14] Another problem is that the Sun is not completely stationary. The Sun and Earth exert equal gravitational forces on each other, but the Sun's greater mass means it accelerates much less. In fact, both objects actually move around their shared center of mass. So, to solve this problem, instead of following each object separately, we can follow them relative to one another using position vectors. This vector points from one body to another, and we define the distance between them as r. What was once a complicated motion of two separate objects can now be treated as a single motion.[15] Another important aspect of orbital motion is the conservation of angular momentum, which quantifies the system's rotation. For an orbit, it depends on the object's position relative to the central body and its motion throughout space. The important thing about angular momentum is that it stays constant in an ideal two-body orbit. The reason comes from the direction of gravity. A force changes angular momentum by creating torque. Gravity acts along the line connecting the two bodies, so it produces no torque about the force center. That said, without external torque, angular momentum is conserved.[16] Connecting back to Kepler, this explains what observers discovered centuries earlier. When a planet gets closer to the Sun, it moves faster; when it moves farther away, it moves slower. Kepler's equal-area law is a consequence of angular-momentum conservation.[17] Conservation of angular momentum also explains why an orbit remains in a single plane. The angular-momentum vector points perpendicular to the plane containing the planet's motion and position. Because that vector remains the same, so does the orbital plane. This matters because instead of describing the planet's motion in three dimensions, we only need to describe it in a two-dimensional plane.

Normally, position in a plane might be shown using Cartesian coordinates, x and y. While useful for many types of motion, they become inconvenient with orbits. Gravity does not depend on how far a planet or object has traveled vertically or horizontally; it depends on the distance from the central body. As a planet moves around the Sun, the direction of gravity rotates as well. Because of this, polar coordinates fit the scenario much better. Polar coordinates use two quantities: r,

the distance from the central body, and θ, the angle around the body. This means that instead of asking for the planet's horizontal and vertical position separately, we can ask a more useful question: How far is the planet from the Sun at a particular angle in its orbit? This question is exactly what the orbit equation will eventually tell us.[18]

Now that we have simplified the problem to two bodies and written it in polar coordinates, we can combine Newton's law of gravitation with conservation of angular momentum. Gravity determines the planet's radial acceleration, while conservation of angular momentum determines how its angular motion changes as its distance changes. The goal is to use these together so time doesn’t appear explicitly. Instead, we want the distance to depend on the angle. To simplify the equation, we use a new variable: $u = 1/r$. This may seem arbitrary, although it is especially useful because gravity itself depends on $1/r^2$. That said, this substitution makes deriving the equation significantly easier. After substituting $u = 1/r$, the equation simplifies down to a second-order differential equation. Solving this equation and changing back to r gives the final result:[19] $r(\theta) = p/[1 + e\cos(\theta - \theta_0)]$. This is the orbit equation.

This equation gives the distance between the two bodies, r, for any position, θ. The constant p sets the orbit's scale and relates to the objects' angular momentum and the strength of gravity. The constant θ_0 determines the orbit's orientation. Most importantly, e represents the orbit's eccentricity, which determines the orbit's shape.[20] Different values of e produce different trajectories. When $e = 0$, it produces a circular orbit. When $0 < e < 1$, it produces an elliptical orbit. When $e = 1$, it produces a parabolic orbit. Finally, when $e > 1$, it produces a hyperbolic orbit.[21] This is what makes the orbit equation so extraordinary. Newton didn't need one equation for planets and another for comets. The same equation produces all of these trajectories. What changes are the object's initial conditions, such as position and velocity.

This equation grew out of ideas hundreds of years old, yet it still plays a vital role today. What began as Newton’s attempt to explain planetary motion has now become a framework for understanding gravity as a whole. This becomes clear

when looking at many modern-day innovations. One example is satellites. Once Newton's laws are used on Earth-orbiting objects, engineers can predict where a satellite will travel as well as determine the speed and altitude required to keep it in orbit. In other words, the same math that can describe planets around the Sun can be applied to artificial objects around Earth. NASA itself notes that modern satellite tracking still relies on Newton's laws of motion.[22] Building on this same foundation, the scope expands from just Earth to the entire Solar System through space missions. When spacecraft travel between planets, it's not as simple as going straight towards the planet. Planets constantly orbit the Sun and move through the universe. That said, engineers use orbital mechanics to create trajectories that use the gravitational forces of other planets.[23] These paths are far more complex than the simple two-body system used in the orbit equation, since real missions must account for far more gravitational forces.[24] Even so, these calculations are still rooted in Newton's framework. The same principles also apply to objects that pose risks to Earth, such as asteroids and comets. Using Newton's laws, NASA can calculate paths and future positions with high precision.[25] This allows scientists to determine if certain objects will require monitoring, as they could pose a risk.[26] However, as powerful and useful as Newton's model is, it is not entirely accurate. When gravitational fields become extremely strong, such as near black holes, Newtonian laws begin to break.[27] In these situations, a more advanced theory is required. Einstein's general theory of relativity replaces the idea that gravity is a force with the concept of curved spacetime. This concept not only provides a more accurate description of motion under extreme conditions such as black holes, but also offers a much better explanation of gravity as a whole.[28] Despite these limitations, the orbit equation remains one of the most useful equations in physics. From everyday technology on Earth to exploration beyond Earth, the same principle connects and explains motion throughout the solar system. The result shows how one relatively simple equation can approximate many orbits in our Solar System, which holds major significance for current-day applications, as well as applications for the future.

Footnotes

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